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Recoil Distributions in Some Proton Reactions

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The distributions of recoil products from the two most important double decays of silicon exicted by proton absorption are calculated by analytical and numerical methods. The nuclear reactions treated are Si(p,pα) and Si(p,2p) and it is assumed the decays are by evaporation. Conversion of the results to distributions of dose is discussed. The latter distributions are of interest in the soft upset of satellite electronic memories.

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RECOIL DISTRIBUTIONS IN SOME PROTON REACTIONS

Single event upset in electronic memory devices has caused a great deal of concern in the last two or three years.[1] One particular area of concern is upset produced in memories on board spacecraft which operate in the earth's radiation belts. Recent NRL calculations have shown[2] that upset rates can be very high in spacecraft memories. Though protons do not deposit energy sufficiently fast to produce upsets directly, some of their secondary products are sufficiently heavy and energetic to do so and occur with sufficient frequency to contribute to observed rates. This paper calculates the energy distribution of the recoil nuclei from the two most important proton induced reactions, Si(p,pα)Mg and Si(p,2p)Al. The dose density from the recoil is easily obtained from this and is briefly discussed. It is assumed for each case that the two light products are emitted essentially simultaneously so that the recoil velocities for both steps of the two step decay are calculated using the mass of the residual nucleus.

Manuscript submitted November 16, 1981.

An excited nucleus with velocity $\vec{v}_0$ decays first with a recoil velocity distribution, $D_1(v_1)$, spherical relative to $\vec{v}_0$, and again with $D_2(v_2)$, also spherical relative to $\vec{v}_1$. In order to calculate the distribution of the energy deposited by the recoil, it is first necessary to obtain the combined distribution of the two steps in the frame of $\vec{v}_1$. Assuming the decays are by evaporation[3]

$$D_i(E_i) = c_i(E_i - B_i)e^{-E_i/t_i}, \quad i = 1,2, \quad E_i \geq B_i \tag{1}$$

where B_i and t_i are reaction constants and c_i is determined by

$$1 = \int dE\, D_i(E). \tag{2}$$

The energy parameter is center of mass reaction energy so

$$E_i = \frac{1}{2} m_i v_i^2,$$

$$m_i = \frac{M_R}{M_i}(M_R + M_i), \quad M_1 = M_p, \quad M_2 = \begin{cases} M_\alpha, \\ M_p \end{cases} \quad M_R = \begin{cases} M(^{24}Mg) \\ M(^{27}Al) \end{cases} \tag{3}$$

Carrying out the normalization, one obtains

$$c_i = t_i^{-2} e^{B_i/t_i}. \tag{4}$$

Since it is our intention to apply the vector convolution theorem in velocity space[4] it is first necessary to change the given distributions from energy densities to velocity space densities.[5] From (2)

$$1 = m \int D(v) v dv$$

$$= (4\pi)^{-1} m \int v^{-1} D(v) d^3v. \tag{5}$$

Thus to convert D to a velocity space density one writes

$$D'(v) = (4\pi v)^{-1} m D(v). \tag{6}$$

Assuming no correlation between D_1 and D_2, the combined distribution is given as a function of $\vec{u} = \vec{v}_1 + \vec{v}_2$ in the same frame as v_1 by the convolution theorem.

$$D'(\vec{u}) = (4\pi)^{-2} m_1 m_2 \iint v_1^{-1} v_2^{-1} D_1(v_1) D_2(v_2) \delta(\vec{u} - \vec{v}_1 - \vec{v}_2) d^3v_1 d^3v_2$$

$$= (4\pi)^{-2} m_1 m_2 \iint v_1^{-1} v_2^{-1} D_1(v_1) D_2(v_2) d^3v_1, \quad v_2 = |\vec{u} - \vec{v}_1|,$$

$$= (4\pi)^{-2} m_1 m_2 \int D_1 D_2 \, v_1 v_2^{-1} \, dv_1 d\mu_1 d\phi_1$$

$$= (8\pi)^{-1} m_1 m_2 \int D_1 D_2 \, v_1 v_2^{-1} \, dv_1 d\mu_1, \tag{7}$$

where $\mu_1 = \cos\theta_1$. Note that $v_2 = 0$ does not occur. Since

$$v_2^2 = u^2 - 2uv_1\mu_1 + v_1^2, \tag{8}$$

the entire μ_1 dependence is in $v_2^{-1}D_2$. Also one has from (8), holding u and v_1 fixed,

$$v_2 dv_2 = -uv_1 d\mu_1$$

$$v_2^{-1} d\mu_1 = -(uv_1)^{-1} dv_2. \tag{9}$$

As μ_1 goes from -1 to 1, v_2 goes from $v^+ = u+v_1$ to $v^- = |u-v_1|$. For the μ_1 integral then one has

$$\int_{-1}^{1} v_2^{-1} D_2 d\mu_1 = -(uv_1)^{-1} \int_{v+}^{v-} D_2 dv_2$$

$$= -(uv_1 t_2^2)^{-1} \int_{v^+}^{v-} (\tfrac{1}{2} m_2 v^2 - B_2) e^{-(\frac{1}{2} m_2 v^2 - B_2)/t} dv$$

$$= -(uv_1)^{-1} (2)^{\frac{1}{2}} (t_2 m_2)^{-\frac{1}{2}} e^{B_2/t_2} \int_{w^+}^{w-} (w^2 - B_2/t_2) e^{-w^2} dw$$

$$= (uv_1)^{-1} (2t_2 m_2)^{-\frac{1}{2}} e^{B_2/t_2} \left[we^{-w^2} \Big|_{w+}^{w-} + (2B_2/t_2 - 1) \int_{w+}^{w-} e^{-w^2} dw \right]$$

$$= (uv_1)^{-1} (2t_2 m_2)^{-\frac{1}{2}} e^{B_2/t_2} \left(we^{-w^2} + \frac{1}{2} (\pi)^{\frac{1}{2}} (2B_2/t_2 - 1) \operatorname{erf}(w) \right) \Big|_{w+}^{w-}. \tag{10}$$

The point u = 0 will not be a problem after changing back to energy density below. Since there is no remaining angular dependence, $D'(\vec{u}) = D'(u)$ and,

$$D'(u) = \frac{m_1}{8\pi u}\left(\frac{m_2}{2t_2}\right)^{\frac{1}{2}} e^{B_2/t_2} \int_{v_m}^{\infty} dv_1 D_1(v_1) F(v_1)$$

where

$$F(v_1) = \left(we^{-w^2} + \frac{1}{2}(\pi)^{\frac{1}{2}} (2B_2/t_2 - 1)\,\mathrm{erf}(w) \right) \Bigg|_{w^+}^{w^-}$$

and

$$w^+ = \left(\frac{m_2}{2t_2}\right)^{\frac{1}{2}} (u+v_1), \quad w^- = \mathrm{Max} \left\{ \left(\frac{m_2}{2t_2}\right)^{\frac{1}{2}} |u-v_1|, \left(\frac{B_2}{t_2}\right)^{\frac{1}{2}} \right\}$$

with

$$v_m = (2B_1/m_1)^{\frac{1}{2}}. \tag{11}$$

For the reactions of interest $B_2 \geq B_1$ and $B_2/m_2 \geq B_1/m_1$ so the lower limit for v_1 includes that for v_2. Also in evaluating (11), whenever $(B_2/t_2)^{\frac{1}{2}} > w^+$, F, and thus the integrand, vanish at this particular v_1.

Of course at this point one has only the spherical distribution relative to $\vec{v}_o$. This is a convenient point at which to check normalization. One requires

$$1 = \int D'(u) d^3u$$

$$= 4\pi M_R^{-1} \int D'(E) u dE \tag{12}$$

so a simple check is just a sum over

$$D(u) = 4\pi u M_R^{-1} D'(u) \qquad (13)$$

on a grid linear in energy.

In the numerical evaluation it was convenient to program D_1 and F as statement functions. The error function (erf) is a single precision library function and ought to be fairly fast. Still each call of F is roughly equivalent to the evaluation of two one-dimensional integrals. Use of Simpson rule is made so one loop for evaluation and one for integration was written to allow printing one complete integrand for error checking. Another convenient error check was to require the sign of $F(v_1)$ to be positive. The constants are $B_1 = 2.4$, $B_2 = 4.6$, $t_1 = t_2 = 2.45$ in MeV. Units of the D_i are MeV^{-1} so writing the masses in MeV gives the velocities in units of light velocity, c. Since the integral is in velocity and the normalization check is in energy, the corresponding loops are on different grids designed to evaluate overlapping regions in $D_1(v_1)$ and D'(u). Thus it was convenient to begin each loop with a unitless energy scale using B_1 as the unit. Then, for example, if a range in E(u) from 0 to 10 (10 B_1 MeV) gives an adequate evaluation of D_1 the appropriate range for v_1/v_m would be 1 to $10^{\frac{1}{2}}$. While in principle there is a serious complication of upper limits required by numerical evaluation, this is ignorable in practice because of the property of these distributions in having compact support. Thus any accuracy desired within computer precision may be attained simply by extending the upper limits sufficiently.

The program RECOIL (listed in the Appendix) has been written, the first half performing the preceding calculations, and the results from this part

are given in Tables I and II. Upper limits for the unitless energy grids were 49 for (p,pα) and 36 for (p,2p), corresponding to evaporation energies of 117.6 and 86.4 MeV. For comparison the peak of the evaporation distribution is at B_i+t_i, or 7.05 and 4.85 MeV, respectively. The overall sums obtained were 0.996 and 0.989, respectively. It may be seen below that extending the energy range decreases this error still.

Consider now the transformation of these distributions to the frame of $\vec{v}_o$. Assume absorption of a 30 MeV proton in the initial excitation, forming a compound nucleus. One has

$$M_p v_p = M_c v_o$$

$$\frac{1}{2} M_p v_p^2 = \frac{1}{2} M_c v_o^2 + E_x \qquad (14)$$

and obtains for the excitation energy

$$E_x = (1 - \frac{M_p}{M_c})T, \quad T = \frac{1}{2} M_p v_p^2 \qquad (15)$$

Taking $M_c = M(^{28}Si+p)$ one obtains $E_x = 28.96$ MeV, $E_c = \frac{1}{2} M_c v_o^2 = 1.04$, $v_o = 0.008945c$. Thus the transformation is in part accomplished by the substitution

$$u^2 = v^2 - 2vv_o\cos\theta + v_o^2, \qquad (16)$$

Table I

PROGRAM RECOIL ON (P,PA)

SPHERICAL ENERGY DISTRIBUTION (PER MEV)

ENERGY	DISTRIB.				
0.0000	0.000000	1.6444	0.429517	3.2888	0.080470
0.0967	0.028580	1.7411	0.418171	3.3855	0.069456
0.1935	0.092224	1.8378	0.402229	3.4822	0.059784
0.2902	0.173525	1.9346	0.382284	3.5789	0.051331
0.3869	0.246142	2.0313	0.359196	3.6757	0.043968
0.4836	0.302075	2.1280	0.333924	3.7724	0.037581
0.5804	0.343079	2.2247	0.307411	3.8691	0.032056
0.6771	0.372533	2.3215	0.280503	3.9659	0.027292
0.7738	0.393493	2.4182	0.253905	4.0626	0.023196
0.8706	0.408342	2.5149	0.228177	4.1593	0.019682
0.9673	0.418841	2.6117	0.203730	4.2560	0.016673
1.0640	0.426263	2.7084	0.180844	4.3528	0.014104
1.1607	0.431509	2.8051	0.159683	4.4495	0.011916
1.2575	0.435221	2.9018	0.140330	4.5462	0.010053
1.3542	0.437758	2.9986	0.122791	4.6430	0.008471
1.4509	0.438514	3.0953	0.107026	4.7397	0.007129
1.5477	0.436139	3.1920	0.092954		

OVERALL SUM 0.996306.

Table II

PROGRAM RECOIL ON (P,2P)

SPHERICAL ENERGY DISTRIBUTION (PER MEV)

ENERGY	DISTRIB.				
0.0000	0.000000	1.0797	0.288428	2.1161	0.003285
0.0432	0.923879	1.1228	0.247268	2.1593	0.002670
0.0864	1.042541	1.1660	0.211004	2.2025	0.002168
0.1296	1.079965	1.2092	0.179297	2.2457	0.001760
0.1727	1.093790	1.2524	0.151765	2.2889	0.001427
0.2159	1.099337	1.2956	0.128065	2.3321	0.001156
0.2591	1.101681	1.3388	0.107606	2.3752	0.000935
0.3023	1.102729	1.3820	0.090182	2.4184	0.000757
0.3455	1.103173	1.4251	0.075367	2.4616	0.000612
0.3887	1.103055	1.4683	0.062821	2.5048	0.000494
0.4319	1.099393	1.5115	0.052236	2.5480	0.000399
0.4750	1.087444	1.5547	0.043337	2.5912	0.000322
0.5182	1.063878	1.5979	0.035878	2.6344	0.000259
0.5614	1.027549	1.6411	0.029644	2.6775	0.000209
0.6046	0.979225	1.6843	0.024448	2.7207	0.000168
0.6478	0.920916	1.7274	0.020128	2.7639	0.000135
0.6910	0.855260	1.7706	0.016544	2.8071	0.000109
0.7342	0.785046	1.8138	0.013578	2.8503	0.000088
0.7774	0.712890	1.8570	0.011127	2.8935	0.000070
0.8205	0.641044	1.9002	0.009107	2.9367	0.000056
0.8637	0.571315	1.9434	0.007444	2.9798	0.000045
0.9069	0.505061	1.9866	0.006077	3.0230	0.000036
0.9501	0.443215	2.0298	0.004955	3.0662	0.000029
0.9933	0.386352	2.0729	0.004036	3.1094	0.000023
1.0365	0.334745				

OVERALL SUM 0.988986

redefining θ as the angle between $\vec{v}_o$ and $\vec{v}$. Similarly defining θ_1, as the angle between $\vec{v}_o$ and $\vec{u}$, one notes that (13) has the form

$$D(u) = \frac{dN}{dE_1 d\Omega_1} \tag{17}$$

where $d\Omega_1 = d\mu_1 \, d\phi_1$, even though it lacks Ω_1 dependence. The tranformation is therefore completed by the Jacobian.

$$\frac{dN}{dE d\Omega} = \frac{\partial(E_1, \Omega_1)}{\partial(E, \Omega)} \frac{dN}{dE_1 d\Omega_1} \tag{18}$$

$$\frac{\partial(E_1, \Omega_1)}{\partial(E, \Omega)} = \frac{v}{u} = \left(\frac{E}{E_1}\right)^{\frac{1}{2}} \tag{19}$$

Calculations of this transformation to the lab have been added to RECOIL, forming the second half. In addition lab solid angle $d\Omega$ is averaged to obtain dN/dE. Further the norm and average energy are obtained. The resulting distributions are in Tables III and IV and Figure 1. These tables show peaks near 1.35 and 0.97 MeV and average energies of 2.55 and 1.55 MeV for (p,pα) and (p,2p) respectively.

Now notice that from (18) one easily obtains an energy deposition distribution, given the recoil energy loss dE/dx,

$$\frac{dN}{dx d\Omega} = \frac{dE}{dx} \frac{dN}{dE d\Omega} \tag{20}$$

If one assumes constant energy loss (a crude approximation) and sets dE/dx = ε, then one recognizes that the multiplication of (18) by a constant is removed by renormalization. One then views (20) as nothing but a unit

Table III

PROGRAM RECOIL ON (P,PA)

SPHERICAL ENERGY DISTRIBUTION:
POINTS: 101 EMAX: 9.6728 DU(EMAX): 0.000000 NORM: 0.999858

LAB ENERGY DISTRIBUTION (PER MEV)

ENERGY	DISTRIB.				
0.0	0.000000	2.7	0.205211	5.4	0.050298
0.1	0.130652	2.8	0.199989	5.5	0.052069
0.2	0.172869	2.9	0.194539	5.6	0.047918
0.3	0.197031	3.0	0.188982	5.7	0.044043
0.4	0.211857	3.1	0.183269	5.8	0.040391
0.5	0.221444	3.2	0.177499	5.9	0.036955
0.6	0.227769	3.3	0.171649	6.0	0.033732
0.7	0.232417	3.4	0.165780	6.1	0.030732
0.8	0.234863	3.5	0.159883	6.2	0.027945
0.9	0.236755	3.6	0.153986	6.3	0.025345
1.0	0.238050	3.7	0.148091	6.4	0.022955
1.1	0.238933	3.8	0.142218	6.5	0.020765
1.2	0.239413	3.9	0.136371	6.6	0.018728
1.3	0.239595	4.0	0.130555	6.7	0.016858
1.4	0.239563	4.1	0.124777	6.8	0.015186
1.5	0.239294	4.2	0.119041	6.9	0.013638
1.6	0.238761	4.3	0.113350	7.0	0.012211
1.7	0.237857	4.4	0.107706	7.1	0.010944
1.8	0.236575	4.5	0.102119	7.2	0.009793
1.9	0.234942	4.6	0.096583	7.3	0.008737
2.0	0.232771	4.7	0.091130	7.4	0.007792
2.1	0.230124	4.8	0.085754	7.5	0.006946
2.2	0.226993	4.9	0.080484	7.6	0.006177
2.3	0.223406	5.0	0.075342	7.7	0.005487
2.4	0.219371	5.1	0.070323	7.8	0.004873
2.5	0.214957	5.2	0.065477	7.9	0.004321
2.6	0.210242	5.3	0.060796	8.0	0.003824

LAB NORM: 9.94159E-01 EAVE: 2.5550

Table IV

PROGRAM RECOIL ON (P,2P)

SPHERICAL ENERGY DISTRIBUTION:
POINTS: 129 EMAX: 5.5278 DU(EMAX): 0.000000 NORM: 0.988988

LAB ENERGY DISTRIBUTION (PER MEV)

ENERGY	DISTRIB.				
0.0	0.000000	1.7	0.352809	3.4	0.063937
0.1	0.157603	1.8	0.332330	3.5	0.053826
0.2	0.231629	1.9	0.312182	3.6	0.044920
0.3	0.287849	2.0	0.292375	3.7	0.037175
0.4	0.334630	2.1	0.272955	3.8	0.030526
0.5	0.374476	2.2	0.253928	3.9	0.024877
0.6	0.407510	2.3	0.235322	4.0	0.020127
0.7	0.431137	2.4	0.217077	4.1	0.016164
0.8	0.439071	2.5	0.199204	4.2	0.012897
0.9	0.442378	2.6	0.181680	4.3	0.010234
1.0	0.442665	2.7	0.164526	4.4	0.008075
1.1	0.441257	2.8	0.147774	4.5	0.006328
1.2	0.438627	2.9	0.131577	4.6	0.004927
1.3	0.431510	3.0	0.116076	4.7	0.003831
1.4	0.412558	3.1	0.101432	4.8	0.002959
1.5	0.393624	3.2	0.087793	4.9	0.002266
1.6	0.373189	3.3	0.075273	5.0	0.001739

LAB NORM: 9.85881E-01 EAVE: 1.5531

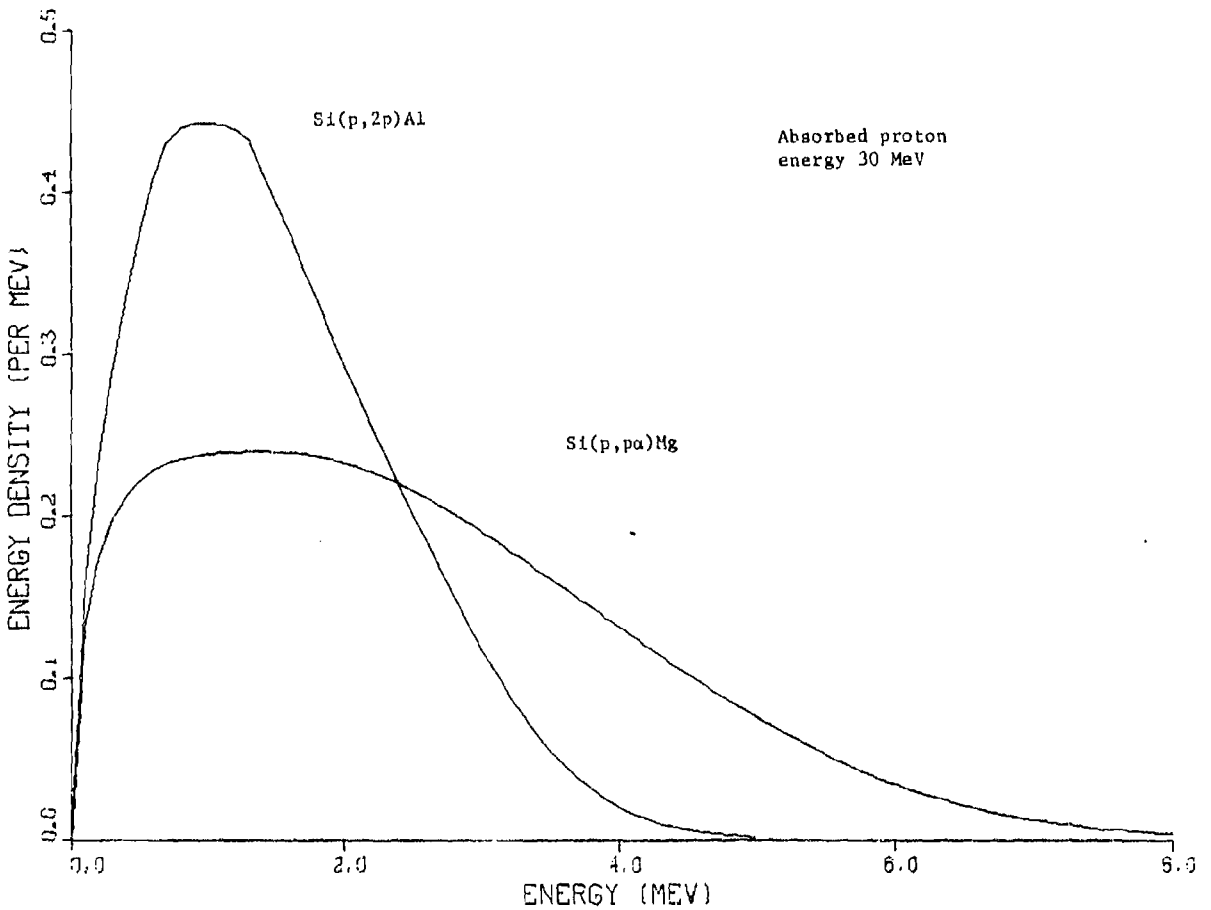

Fig. 1 — Recoil distributions

change and accepts the numerical distribution (18) as giving the distribution per ε^{-1} μm, the distance in which 1 MeV is lost. Instead of assuming constant energy loss one may utilize the code E-DEP-1[6] to include straggling effects. A minor change will be required however since the energy-deposited report of E-DEP-1 does not include ionization loss, a significant part of dose. In making this change it will be permissible to ignore radiation loss, a few percent effect. Short of using E-DEP-1 but better than assuming constant dE/dx, one can find expressions giving the variation.

In fact $\varepsilon \approx 1$ MeV/μm is only about twenty percent low when averaged over the first few μm. Thus the implication of Figure 1 is that significant energy, deposited in a spherical region of radius 3 to 4 μm, should be added to that previously considered.

Acknowledgments

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APPENDIX

SOURCE LISTING ASC FAST FORTRAN COMPILER

CSN STATEMENT CP OPTIONS = (M,X)

```
0001            PROGRAM RECOIL
                C SET UP FOR SI(P,P ALPHA).  FOR (P,2P) SET B2=B1, MA=MP & CHANGE MR
                C TRANSFORMATION OF SPHERICAL DISTRIBUTION TO LAB ADDED
0002            IMPLICIT REAL(M)
0003            DIMENSION SG(241),DU(82),EU(82),DV(66),EV(66)
0004            DIMENSION DV2(22),EV2(22),DV3(22),EV3(22)
0005            EQUIVALENCE (DV2,DV(23)),(EV2,EV(23)),(DV3,DV(45)),(EV3,EV(45))
0006            DATA MC,MP,MA,MR/931.5016,938.2796,4.0026,23.98504/,B1,B2,T
                */2.4,4.6,2.45/,DE/1.0/,A/0./,I1,J1,K1/82,241,66/,MT/27.97693/
0007            E1(V)=(V*V*M1-B1)*RT
0008            D1(V)=RT*E1(V)*EXP(-F1(V))
0009            FW(V)=V*EXP(-V*V)+FC*ERF(V)
0010            J2=J1-1
0011            DV1=8./FLOAT(J2)
0012            MA=MA*MC
0013            MR=MR*MC
0014            M0=0.5*(MR+MP)/MP
0015            M1=M0*MR
0016            M2=MR*(MR+MA)*0.5/MA
0017            RT=1.0/T
0018            VW=SQRT(M2*RT)
0019            F=M0*VW*EXP(B2*RT)
0020            P4=ATAN(1.0)
0021            FC=(2.0*B2*RT-1.0)*SQRT(P4)
0022            VU=SQRT(B1/M1)
0023            S1=2.*DV1*VU/3.
0024            VT=SQRT(B2*RT)
0025            PRINT 61
0026      61    FORMAT(' PROGRAM RECOIL ON (P,PA)')
0027            DU(1)=0.
0028            EU(1)=0.
0029            DO 1 I=2,I1
0030            U=SQRT(DE*(I-1))*VU
0031            EU(I)=0.5*MR*U*U
0032            DO 2 J=1,J1
0033            V1=VU*(1.0+DV1*(J-1))
0034            SG(J)=0.
0035            VP=(V1+U)*VW
0036            IF(VP.LE.VT) GOTO 2
0037            VM=ABS(V1-U)*VW
0038            IF(VM.LT.VT) VM=VT
0039            SG(J)=FW(VM)-FW(VP)
0040            IF(SG(J).GT.-1E-6) GOTO 8
0041            PRINT 69,SG(J),V1,U,J,I
0042      69    FORMAT(' ***NEGATIVE INTEGRAND,V1,U,J,I:',1P3E11.2,2I4)
0043      8     SG(J)=F*D1(V1)*SG(J)
0044      2     CONTINUE
0045            S=0.5*(SG(J1)-SG(1))
0046            DO 4 J=2,J2,2
0047      4     S=S+2.*SG(J)+SG(J-1)
```

```
CSN             STATEMENT                       CP OPTIONS = (M,X)

0048                DU(I)=S*S1
0049          1     A=A+DU(I)
0050                A=A*EU(2)
0051                PRINT 62,I1,EU(I1),DU(I1),A
0052          62    FORMAT(//"   SPHERICAL ENERGY DISTRIBUTION:"/"   POINTS:",I4,
                   *"  EMAX:",F7.4,"  DU(EMAX):",F9.6,"  NORM:",F9.6)
0053                MT=MT*MC
0054                ME=MP+MT
0055                EP=30.*MP/ME*MR/ME
0056                EV(1)=0.
0057                DV(1)=0.
0058                DO 10 K=2,K1
0059                EV(K)=0.1*(K-1)
0060                E2=2.0*SQRT(EV(K)*EP)
0061                S=0.
0062                DO 11 J=1,100
0063                I0=1
0064                MU=1.0-(2*J-1)/100.
0065                E0=EV(K)+EP-E2*MU
0066                F1=SQRT(EV(K)/E0)
0067                DO 14 I=I0,I1
0068                IF(E0.LT.EU(I)) GOTO 15
0069          14    CONTINUE
0070                GOTO 16
0071          15    I0=I-1
0072                D=DU(I0)+(DU(I)-DU(I0))*(E0-EU(I0))/(EU(I)-EU(I0))
0073                S=S+D*F1
0074          11    CONTINUE
0075          16    DV(K)=S*0.01
0076          10    CONTINUE
0077                PRINT 64
0078          64    FORMAT(///"   LAB ENERGY DISTRIBUTION (PER MEV) ",
                   *///"      ENERGY DISTRIB.")
0079                PRINT 65,(EV(K),DV(K),EV2(K),DV2(K),EV3(K),DV3(K),K=1,22)
0080          65    FORMAT(3(F10.1,F10.6))
0081                S=0.
0082                A=0.
0083                DO 20 K=1,K1
0084                S=S+DV(K)
0085          20    A=A+(EV(K)+0.05)*DV(K)
0086                A=A/S
0087                S=S*EV(2)
0088                PRINT 66,S,A
0089          66    FORMAT(//"   LAB NORM:",1PE13.5,"  EAVE:",0PF7.4)
0090                END
```